Torque Measurements Using Strain Gauges on the Magnetorotational Instability Experiment

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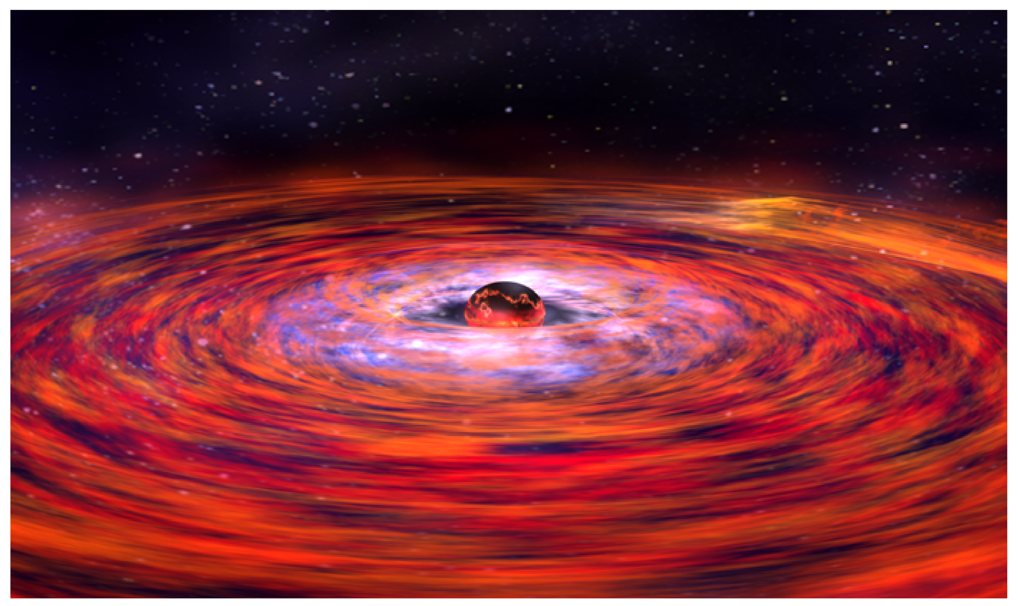
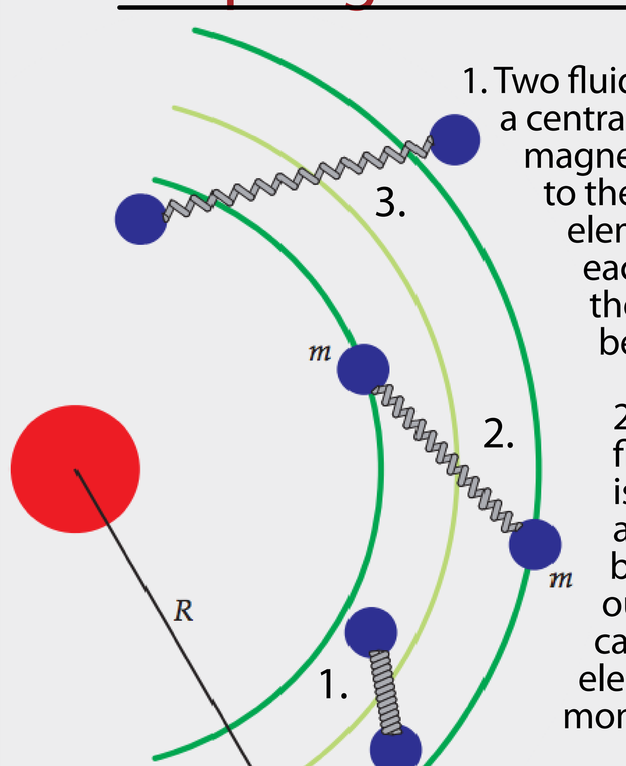
The Magnetorotational Instability and the MRI Experiment at PPPL



A typical accretion disk: diffuse material orbits a massive central body. Disks can form around stars and black holes; and accrete material in jets. A protoplanetary disk is also an accretion disk.

Here, results are presented from the development of strain gauge torque diagnostic designed to measure the effects of the MRI in a laboratory experiment.

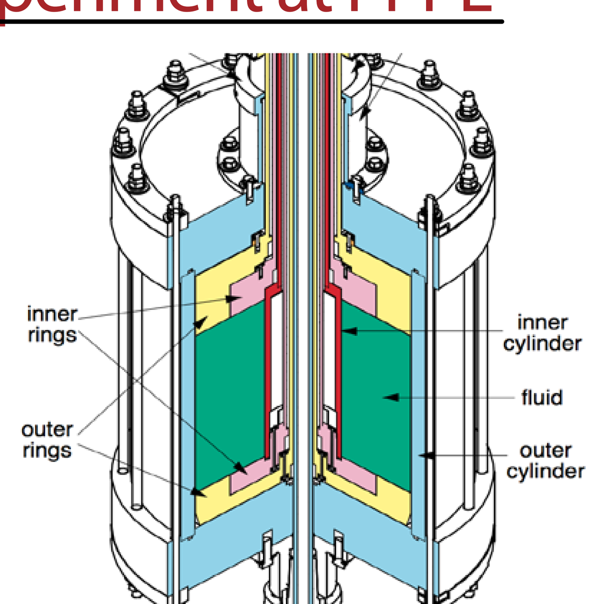
A Spring Model of the MRI



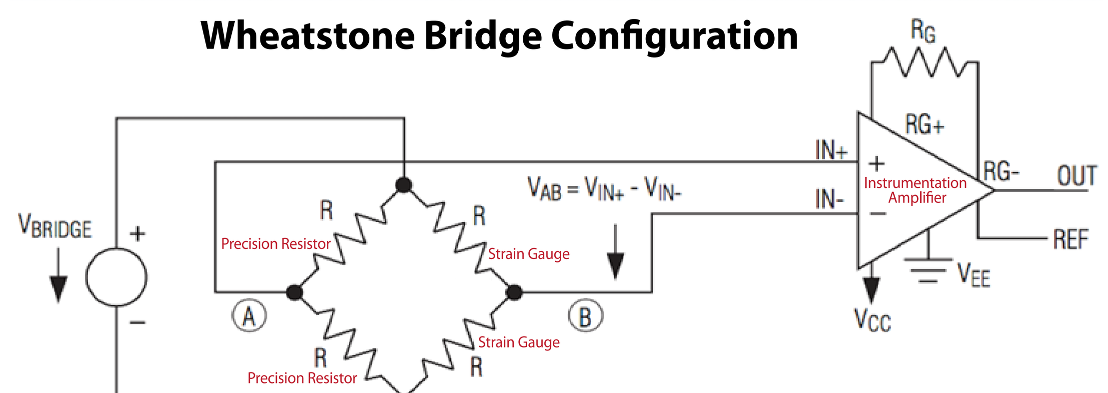
1. Two fluid elements are circling a central massive body. For a magnetic field perpendicular to the page, the two elements are attracted to each other and it is as if there is a spring between them.
2. As in a Keplarian flow, the Inner element is rotating more quickly and will be pulled back towards the outer element. This causes the inner element to lose angular momentum and fall in.
3. The outer element acquires angular momentum and is thrown out of the system.

The MRI Experiment at PPPL

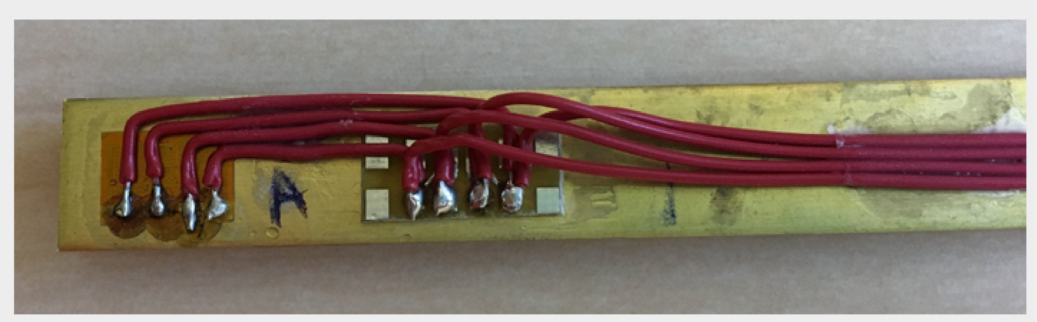
- The working fluid in the MRI apparatus is an alloy of Gallium, Indium and Tin (GalnSn or Gallinstan). It is liquid at room temperature, and is a Eutectic alloy.
- Motors can run the inner and outer cylinders at different speeds creating stable and unstable conditions.
- Diagnostics include, Hall sensors and Ultra Doppler Velocimetry (UDV).

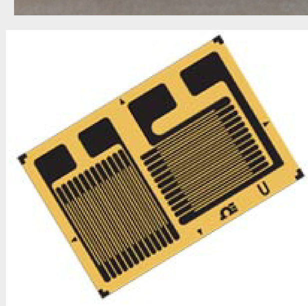


Strain Gauge Diagnostic Set-Up



- Schematic of the Wheatstone bridge configuration used
- Strain gauges made up a half bridge helping to amplify signals received
- In addition, an instrumentation amplifier was used at a gain of ~ 1,000
- Precision resistors were used to complete bridge
- Power supply with two separate outputs were used to supply the bridge with 2.00 - 5.00 Volts and the amplifier with 2.70 volts



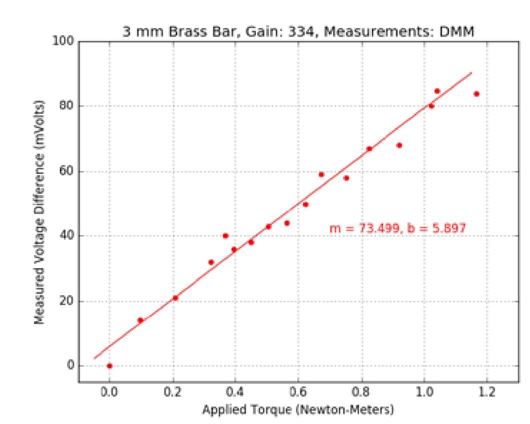


Above: Shows the process of making a strain gauge. The Brass bar measures approx. 12.7 mm wide, and 1.63 mm thick. On the end of the bar is an Omega strain gauge.

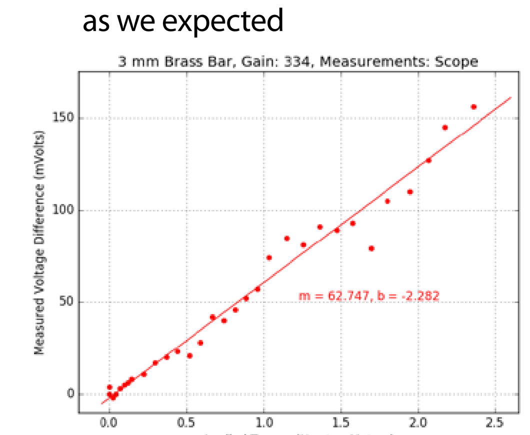
Left: An image of the Omega strain gauges, measuring approx. 7.5 mm x 10.8mm.

Results and Conclusions

Initial Proof of Concept Results:

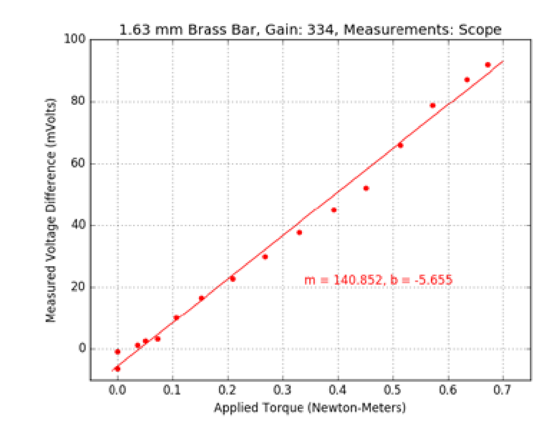


- Strain gauges were attached to a 3 mm thick brass bar in the Wheatstone Configuration
- Results confirmed a linear trend, but suggested amplification was necessary to measure the expected small torques
- Added an instrumentation amplifier to look at the difference in voltage between two sides of the bridge
- Made new measurements on a scope with an averaging function
- Scope results had linear dependence as we expected

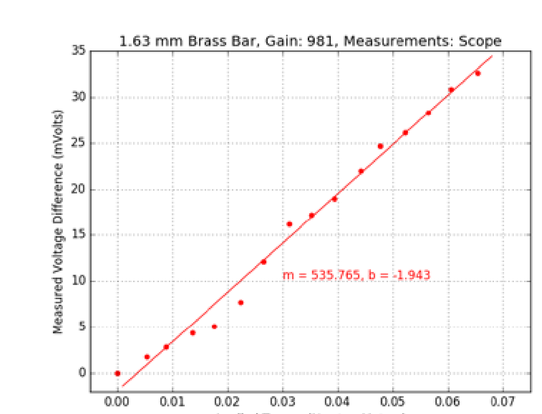


Accurate Parameter Regime Results:

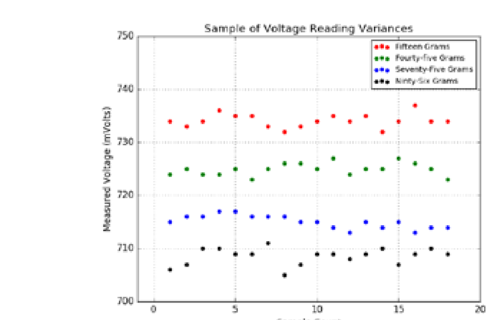
- Constructed new brass bars with half the thickness as previously tested
- Same tests were carried out with smaller torques on the bars
- Results very strongly followed a linear trend again



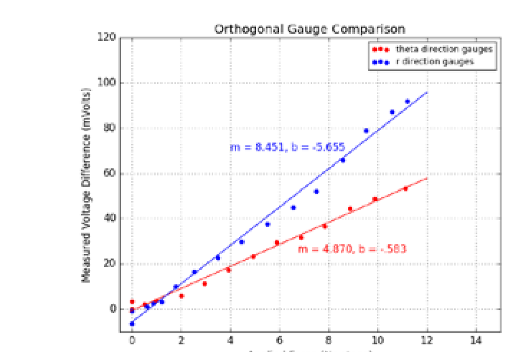
- Increased the gain to be able to be in the regime of the MRI Experiment
- Expect to be in the tens of milli-Newton- meter scale for torques in the MRI experiment.



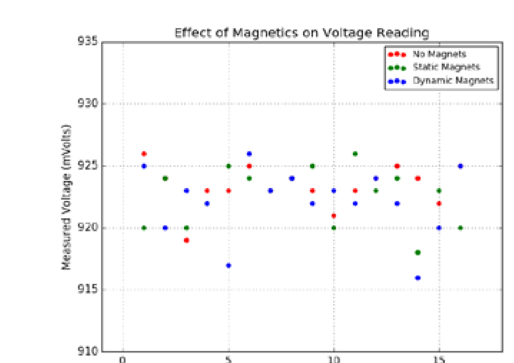
Other Testing Results:



Verifying that an average of readings was a valid approach based on the oscillatory nature of the readings about a mid-value.

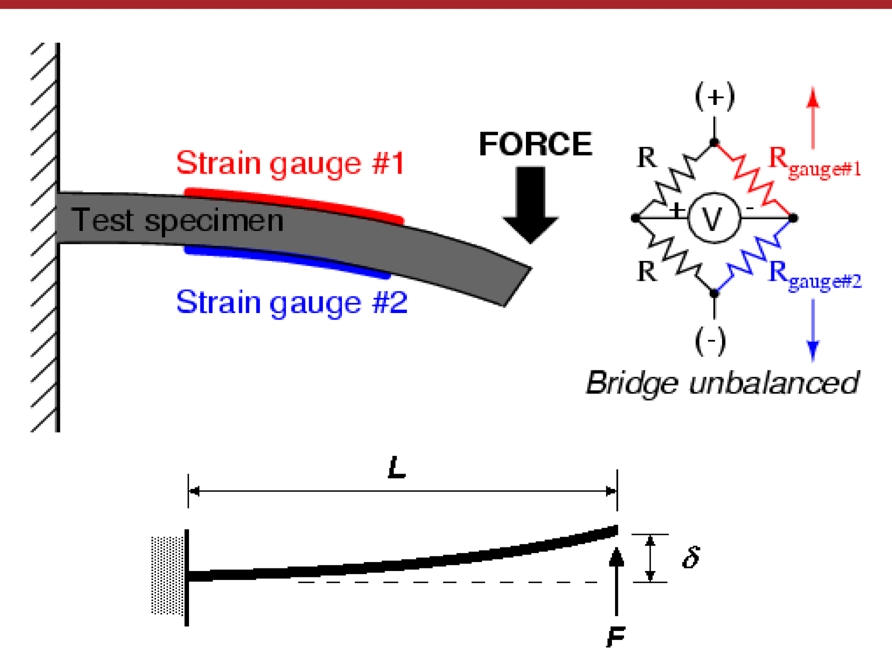


The strain gauges that were perpendicular to the main gauges were thought to read about a constant torque, but didn't.



A preliminary look at the affects of magnetics on the strain gauge show no affect.

Theory for Using Strain Gauges to Measure Torques



- A force deflects the bar
- This causes a change in the length of the strain gauges which registers as a change in their resistance
- This change causes the bridge to become unbalanced
- This is read as a voltage difference between sides of the bridge
- The change in voltage should depend linearly on the torque
- Because of this, we were able to calibrate the circuit and characterize the data by 2 parameters, m and b

Voltage Ratio as a Function of Resistance:

$$\frac{V_{out}}{V_{in}} = \frac{R_{SG_1}}{R_{SG_1}+R_{SG_2}} - \frac{R_2}{R_1+R_2}$$

Relationships Between Resistance, Length and Force:

$$R = \frac{\rho L_{SG}}{A} \qquad \delta = \frac{FL^3}{3EI} \qquad I = \frac{1}{12} w \cdot h^3$$

E = Young's Modulus, w = width of bar, h = height of bar

Linear Relationship Between Torque and Voltage:

$$V \propto R \propto L_{SG} \propto \delta \propto F \propto \tau$$

Thus,

$$\boxed{V = m \cdot \tau + b}$$

Estimated Torque in a Taylor Couette Geometry Using Typical MRI Parameters

Define an Ideal Taylor-Couette Profile:

$$\alpha = \frac{r_2^2 \cdot \Omega_2 - r_1^2 \cdot \Omega_1}{r_2^2 - r_1^2} \qquad \beta = r_1^2 \cdot r_2^2 \left(\frac{\Omega_1 - \Omega_2}{r_2^2 - r_1^2}\right)$$

$$V_\theta(r) = \alpha \cdot r + \frac{\beta}{r}$$

Compute the Torque per unit Height:

$$\tau(r) = -2\pi\nu\rho \cdot r^3 \cdot \frac{\partial}{\partial r}\left(\frac{V_\theta(r)}{r}\right)$$

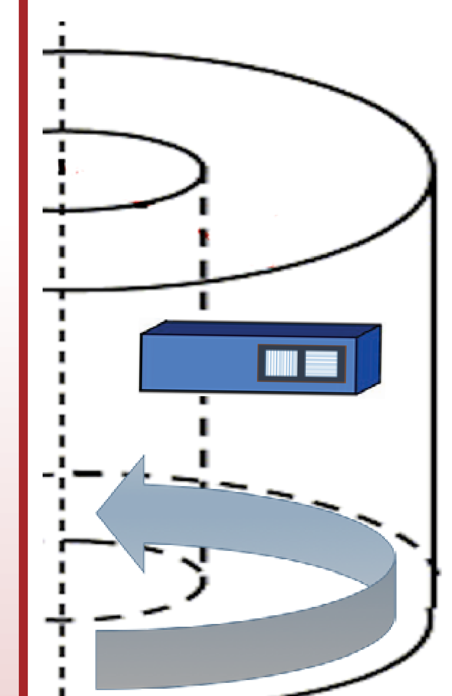
Estimated Torque per unit Height for the MRI Parameters:

$$\boxed{\tau(r_1) = 4.997 * 10^{-3} N}$$

Typical Run Parameters for MRI Experiment

r_1	Inner Radius	.07 m
r_2	Outer Radius	.21 m
Ω_1	Inner Cylinder Speed	43.80 $\frac{rad}{sec}$
Ω_2	Outer Cylinder Speed	5.81 $\frac{rad}{sec}$
ρ	Density of Working Liquid	6.36 $\frac{kg}{Liter}$ (GaInSn)
ν	Kinematic Viscosity	$2.98 * 10^{-7} \frac{m^2}{sec}$

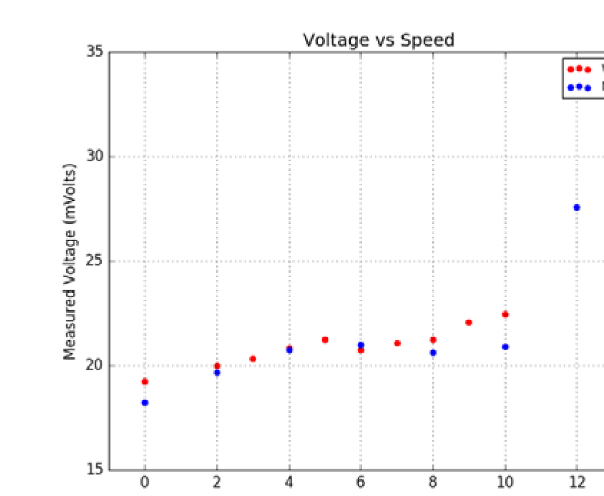
What is Next?



Future Set-Up

- Move the Strain Gauges into the water and MRI experiments
- Strain Gauges will be attached to the inner cylinder
- They will support a sheath of thin material that is coaxial to the existing cylinders and free to move with the fluid.
- The gauges will then measure the torque on this sheath

When the MRI is present it will cause angular momentum transport radially outward from the bulk of the fluid. The angular momentum will be carried in towards the inner cylinder and out towards the outer cylinder. This change in angular momentum will cause torques in the azimuthal direction that can be measured with the strain gauges.



Initial results with testing the strain gauges in a spinning liquid. The preliminary results are very promising, and there is a clear increasing trend of measured voltage with speed of the spinning liquid.

References

1. Ji, H., & Balbus, S. (August 2013). **Angular momentum transport in astrophysics and in the lab.** Physics Today.

2. Lathrop, D., Fineberg, J., & Swinney, H. (1992). **Transition to Shear Driven Turbulence in Couette-Taylor Flow.** Physical Review A.

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